CMSC414 Computer and Network Security

Public Key Cryptography

Yizheng Chen | University of Maryland
surrealyz.github.io

March 31, 2026

Credits: original slides from Dave Levin and CS161 staff at UC Berkeley

Agenda

- Diffie Hellman Key Exchange
- Public Key Cryptography
- Certificates
- Passwords

DIFFIE HELLMAN KEY EXCHANGE

HIGH-LEVEL REVIEW OF MODULAR ARITHMETIC

$x \bmod N$

g is a generator of mod N if
$\{1, 2, \ldots, N\text{-}1\} = \{g^0 \bmod N, g^1 \bmod N, \ldots, g^{N-2} \bmod N\}$

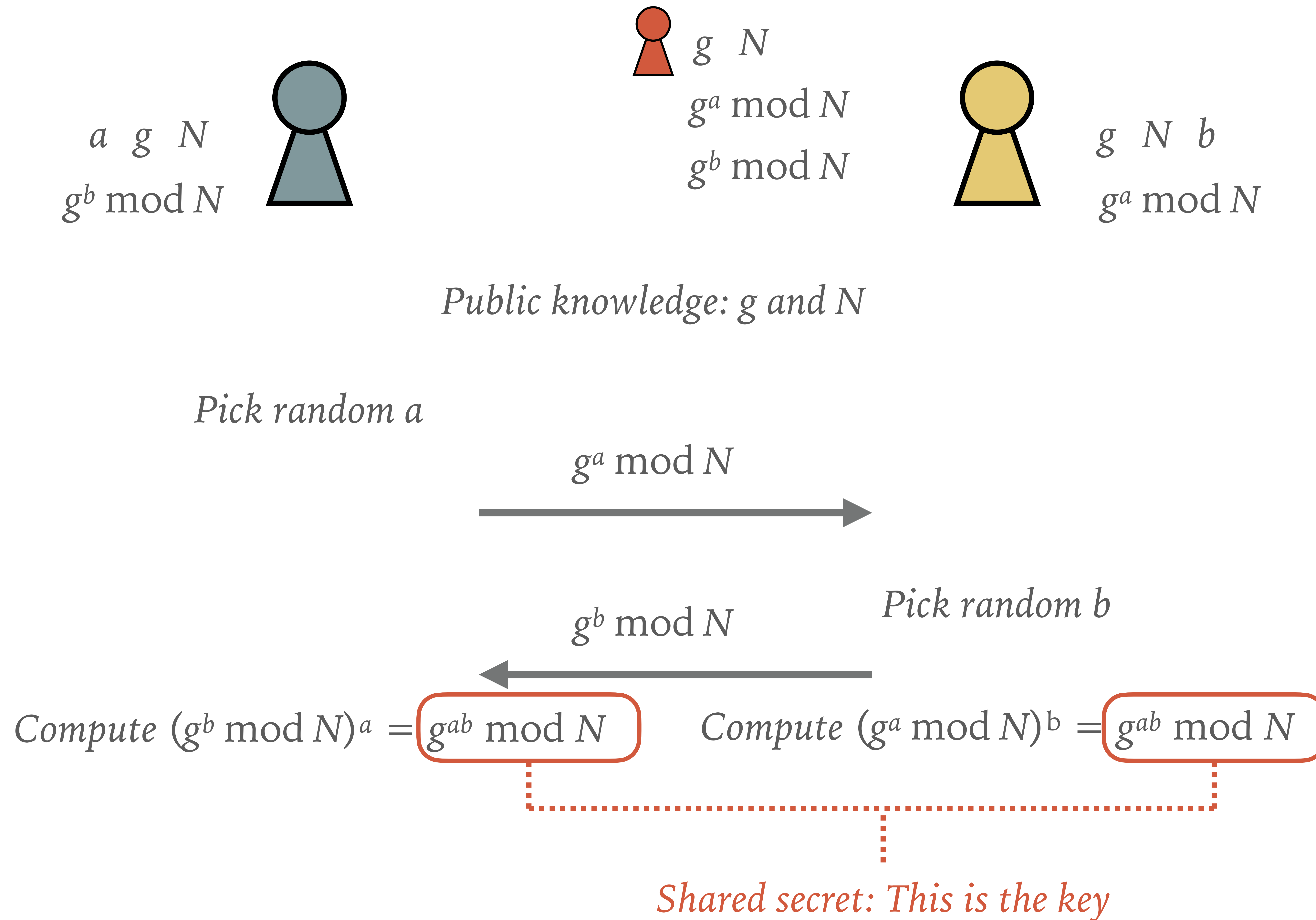
$N=5, g=3$
$3^0 \bmod 5 = 1$ $3^1 \bmod 5 = 3$ $3^2 \bmod 5 = 4$ $3^3 \bmod 5 = 2$

Given x and g, it is efficient to compute
$g^x \bmod N$

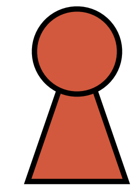
Given g and g^x, it is efficient to compute x
(simply take $\log_g g^x$)

Given g and g^x mod N it is *infeasible* to compute x
Discrete log problem

DIFFIE-HELLMAN KEY EXCHANGE



DIFFIE-HELLMAN KEY EXCHANGE



$g \quad N$

$g^a \bmod N$

$g^b \bmod N$

$g^{ab} \bmod N$

Given g and $g^x \bmod N$ it is *infeasible* to compute x
Discrete log problem

Note that just multiplying g^a and g^b won't suffice:

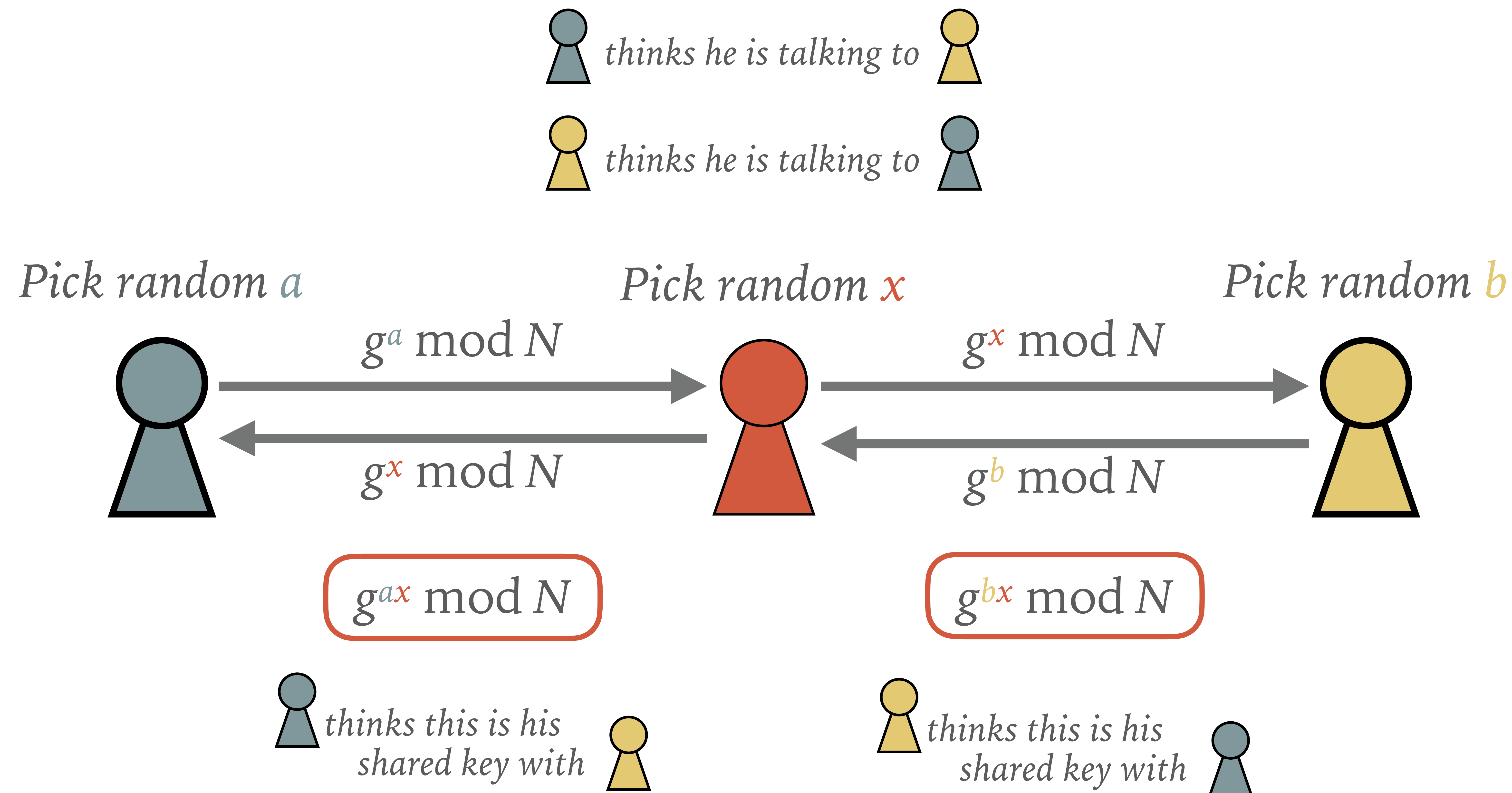
$$g^a \bmod N \ * \ g^b \bmod N \quad = g^{a+b} \bmod N$$

Key property:
An *eavesdropper* cannot infer the shared secret (g^{ab}).

But what about *active intermediaries*?

MAN-IN-THE-MIDDLE (MITM) ATTACKS

The attacker can interpose between the two communicating parties and insert, delete, and modify messages.



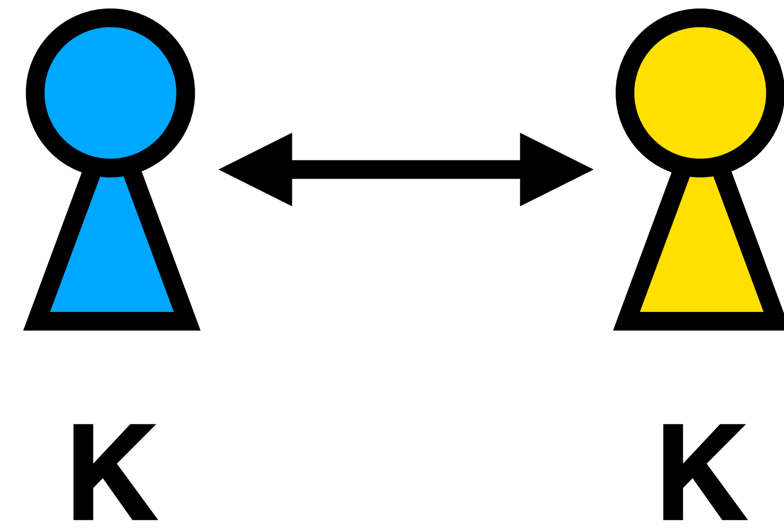
The attacker can now eavesdrop on the conversation.

Key property: Diffie-Hellman is *not* resilient to a MITM attack

TO FIX THIS PROBLEM WE NEED…

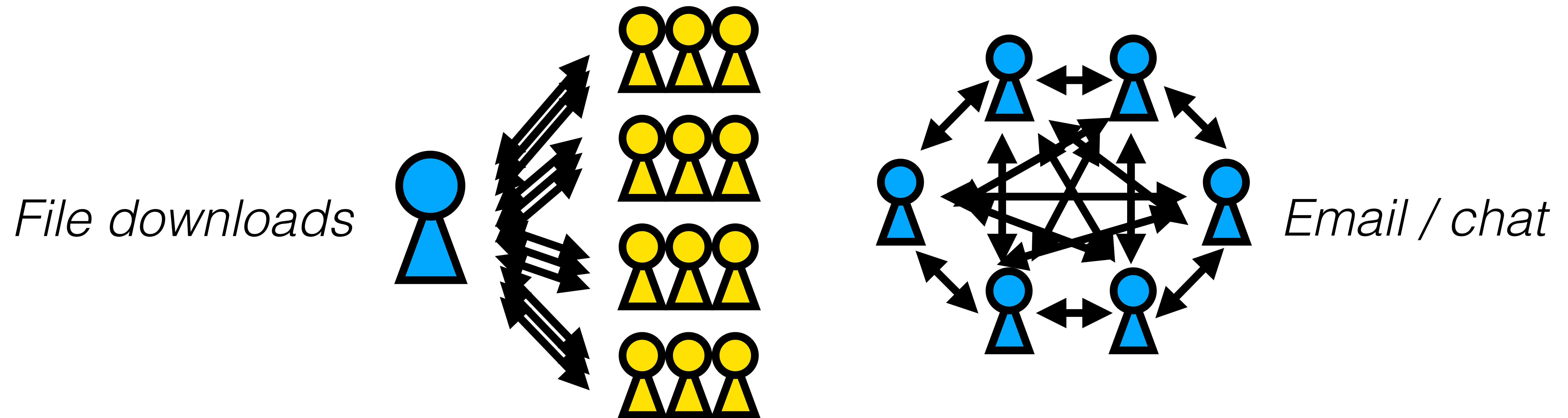
PUBLIC KEY CRYPTOGRAPHY

Shortcomings of symmetric key



Establishing a pairwise key requires a **key exchange**, which requires both parties to be ***online***

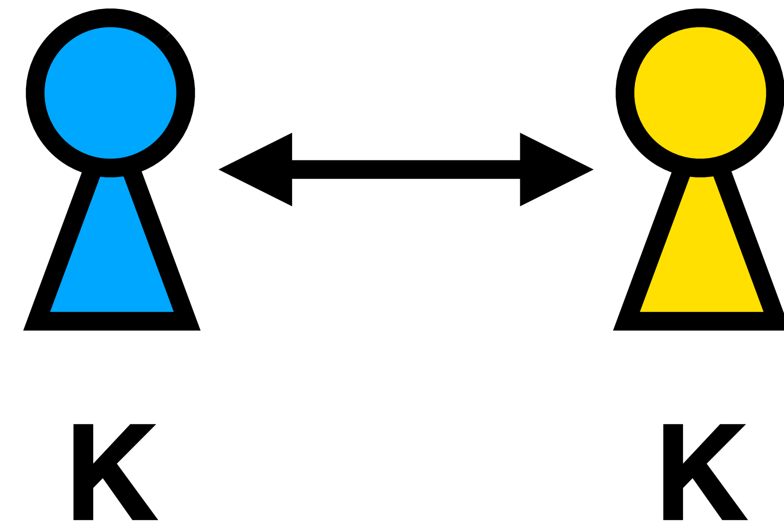
Issue #1: Requires *pairwise* key exchanges



One-to-many: O(N) key exchanges

All-to-all: $O(N^2)$ key exchanges

Shortcomings of symmetric key



Establishing a pairwise key requires a **key exchange**, which requires both parties to be ***online***

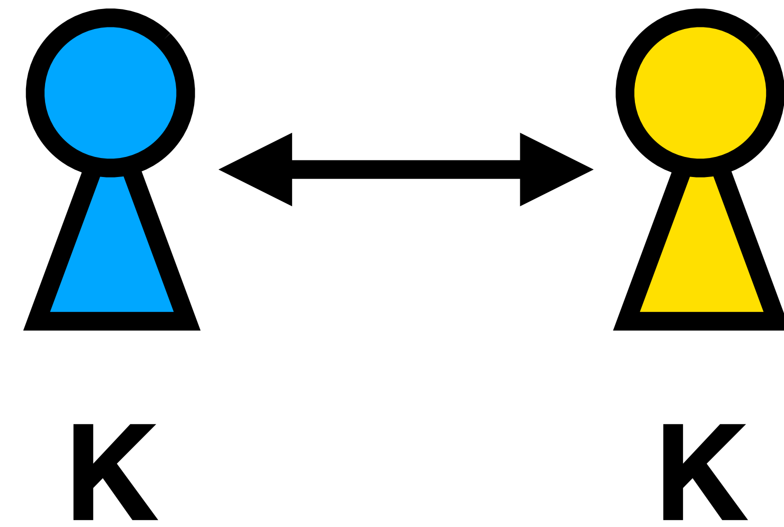
Issue #2: Parties must be online

One-to-many: O(N) key exchanges

Blue user uploads a document, then goes offline (e.g., forever)

Later, a yellow user wants to get a copy; how can it know the copy is really from the blue user?

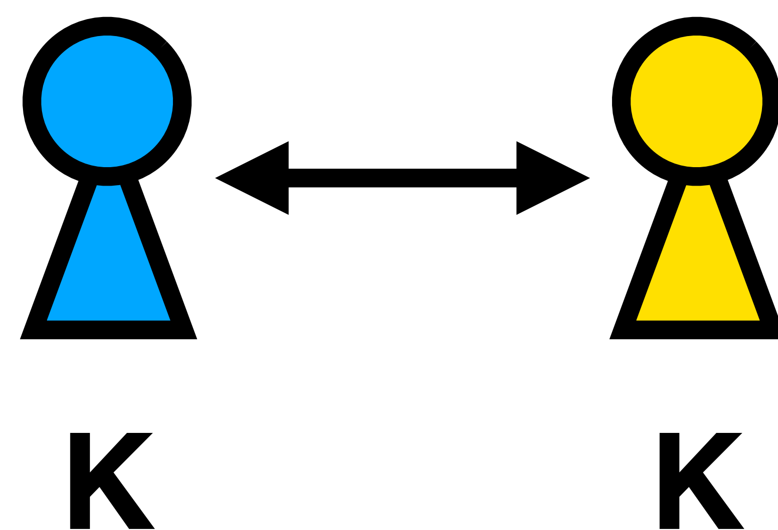
Shortcomings of symmetric key



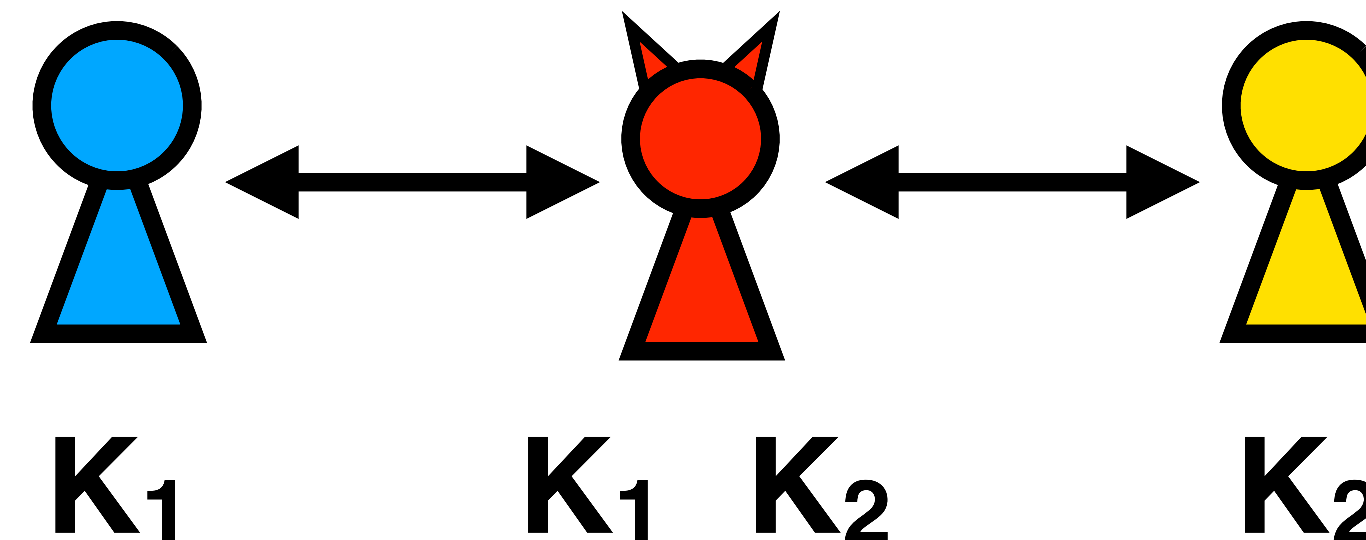
Establishing a pairwise key requires a **key exchange**, which requires both parties to be ***online***

Issue #3: How do you know to whom you're talking?

Diffie-Hellman is resilient to *eavesdropping*, but ***not tampering***

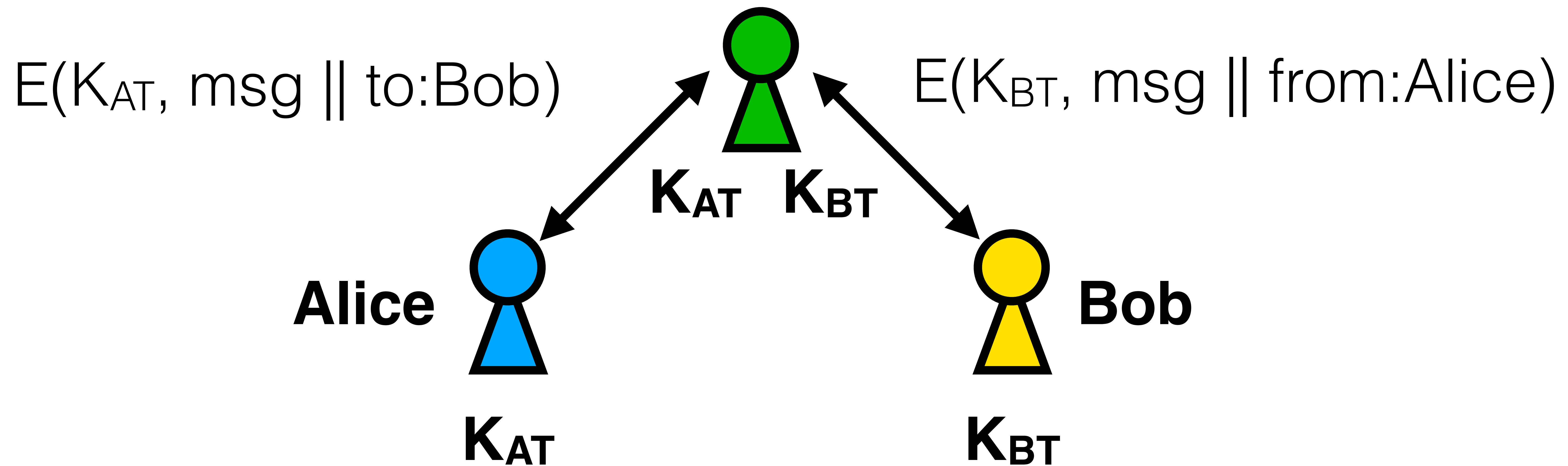


vs



A protocol that solves this with ***trust***

Trent: A *trusted* third party



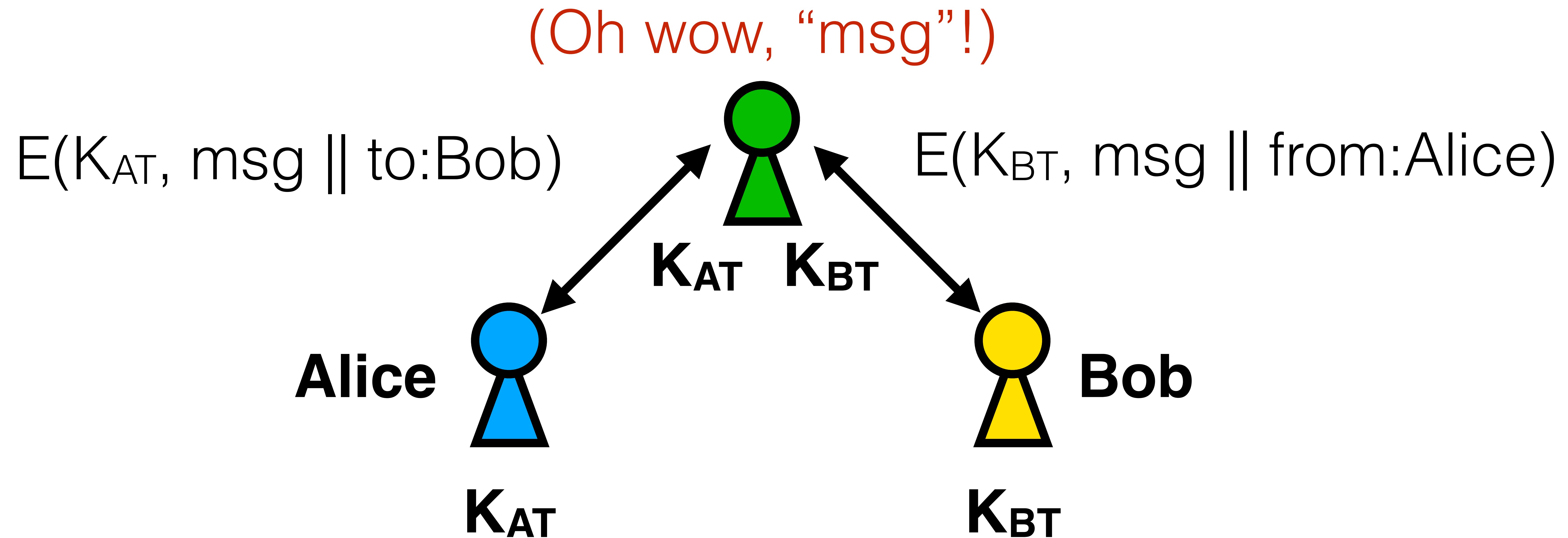
1. Everybody establishes a pairwise key with Trent
Good: *O(N) key exchanges*

2. Trent validates each user's identity; includes in message
Good: *Authenticated communication*

Bad: All messages get sent through Trent

What are we trusting Trent not to do?

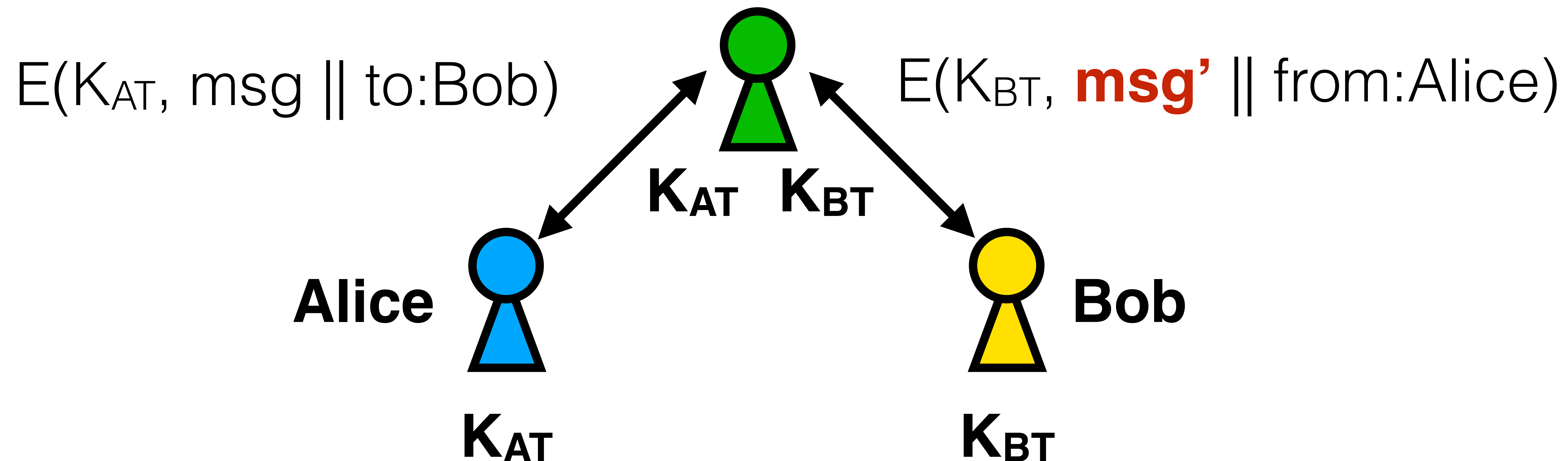
Just as "secure" meant nothing without an attack model,
"trusted" means nothing without a **trust model**



1. Do not *read* messages

What are we trusting Trent not to do?

Just as "secure" meant nothing without an attack model,
"trusted" means nothing without a **trust model**

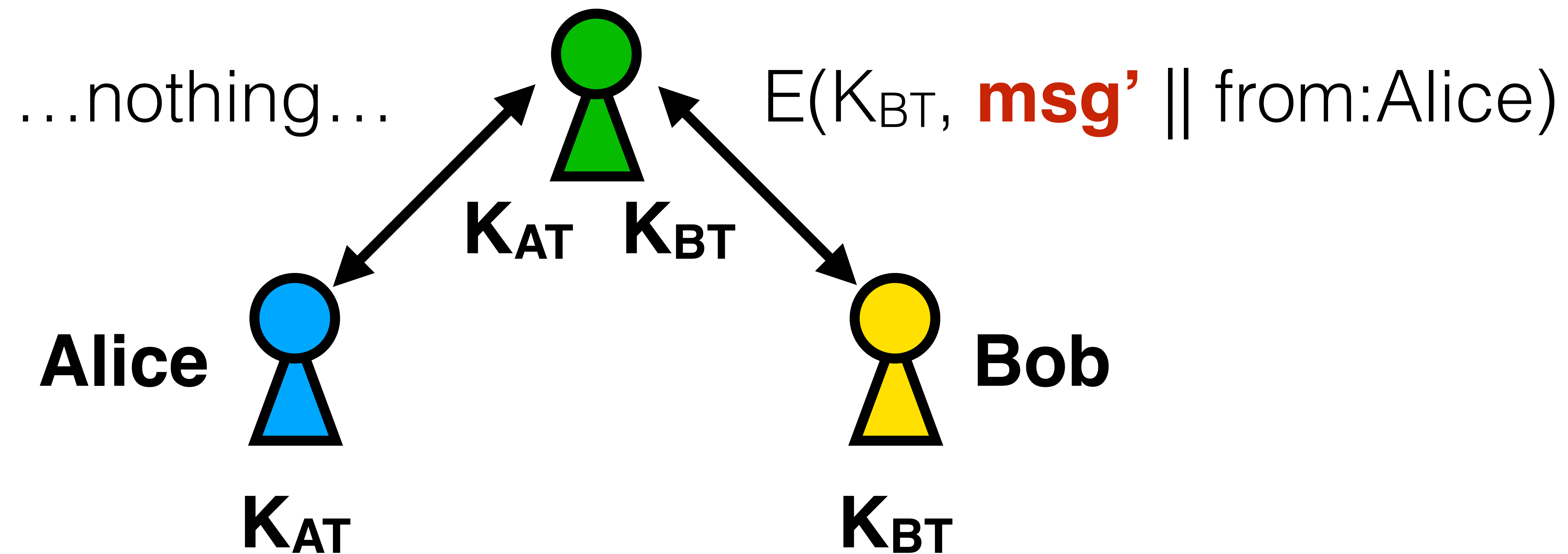


1. Do not *read* messages

2. Do not *alter* messages

What are we trusting Trent not to do?

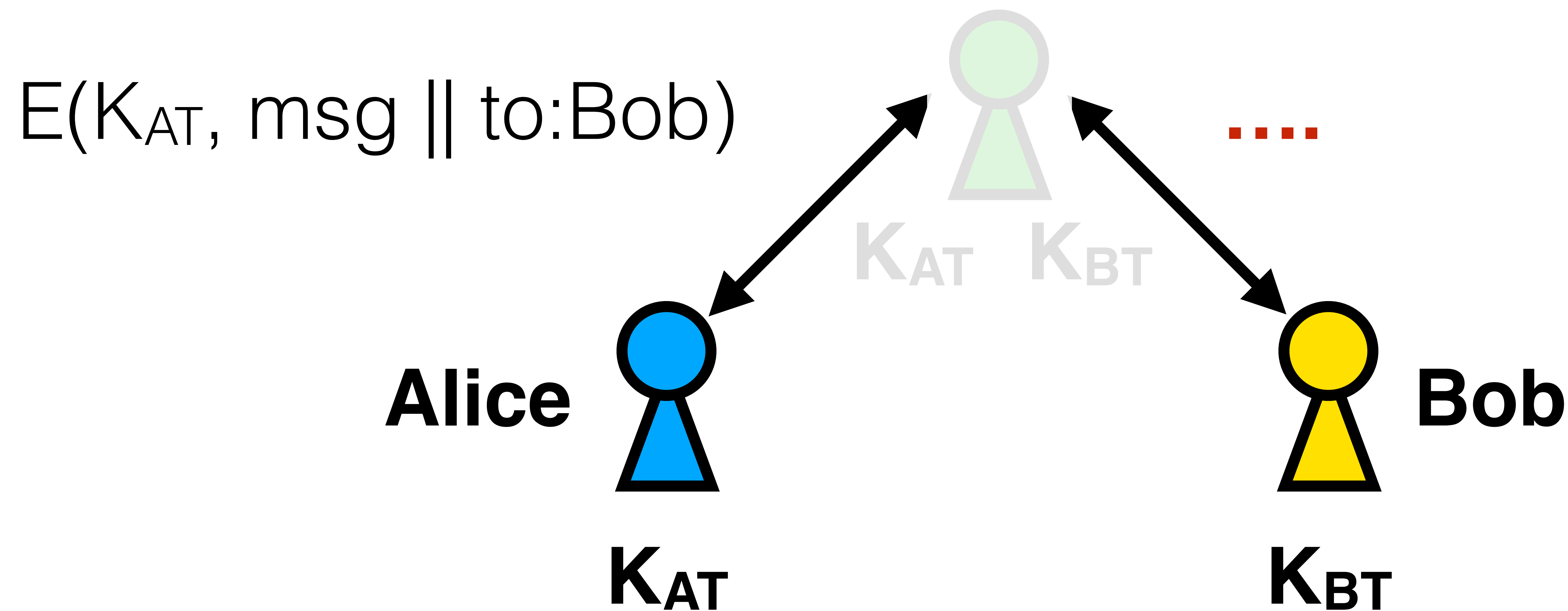
Just as "secure" meant nothing without an attack model,
"trusted" means nothing without a **trust model**



1. Do not *read* messages
2. Do not *alter* messages
3. Do not *forge* messages

What are we trusting Trent not to do?

Just as "secure" meant nothing without an attack model,
"trusted" means nothing without a **trust model**



1. **Do not *read* messages**
2. **Do not *alter* messages**
3. **Do not *forge* messages**
4. **Do not *go offline***

Public key encryption

A public key encryption scheme comprises three algorithms

Key generation **G**

- Inputs
  - Source of randomness
  - Maximum key length L
- Outputs: a *key pair*
  - *PK* = **public key**
  - *SK* = **secret key**

This is a *randomized* algorithm
(nondeterministic output)

Difficult to infer SK from PK
Only one person should know SK;
PK should be public to *all*

PK and SK are intrinsically bound together:
for a given PK, there is a single *corresponding* SK

Example: RSA’s public keys are a pair: (exponent, modulus)

Public key encryption

A public key encryption scheme comprises three algorithms

Encryption **E(PK, msg)**

- Inputs
  - **Public** key PK
  - Message msg of *fixed size*
- Outputs: a cipher text *c* *same size as msg*

This is a *randomized* algorithm

(vanilla RSA is deterministic;
in practice, RSA-PKCS is used instead, which adds a nonce to the message)

PK a.k.a. "Encryption key"

Anyone who knows Alice's PK can encrypt a message to her…

Public key encryption

A public key encryption scheme comprises three algorithms

Decryption **D(SK, c)**
- Inputs
  - **Secret** key SK
  - Cipher text c
- Outputs: original msg

This is a *deterministic* algorithm
Should always return the original message

…but only Alice can decrypt that message

Public key encryption

A public key encryption scheme comprises three algorithms

Key generation G
→ *PK* = **public key**
→ *SK* = **secret key**

Encryption E(PK, m)
→ cipher text *c*

Decryption D(SK, c)
→ original msg

Correctness

D(SK, E(PK, m)) = m

Security

E(PK, m) should appear random (small change to (PK,m) leads to large changes to c)

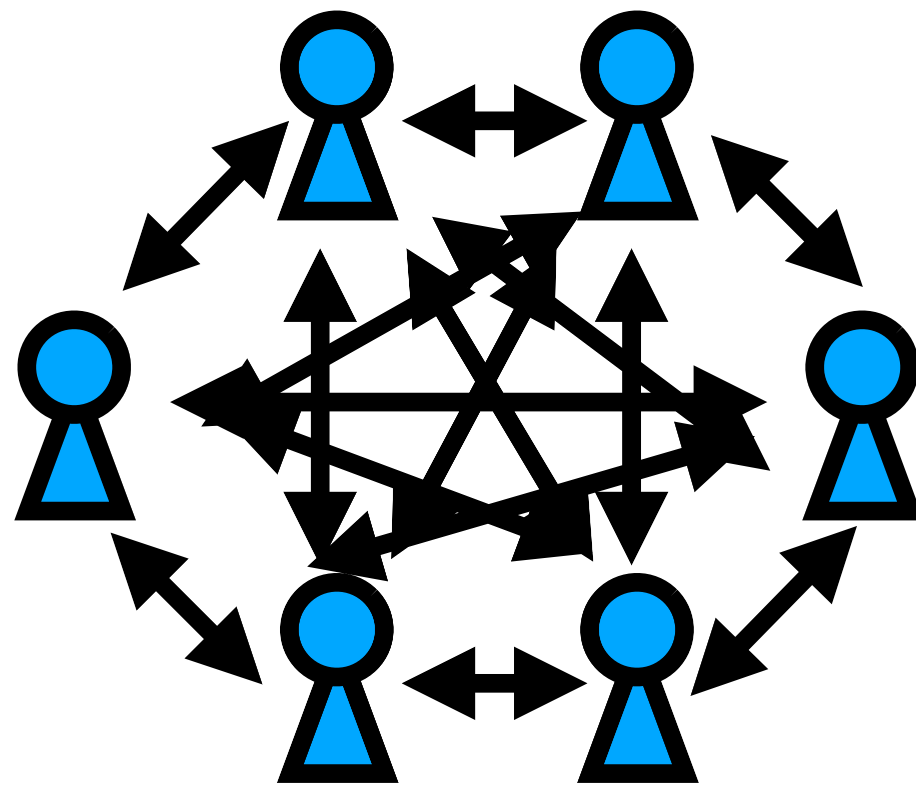
E() should approximate a one-way trapdoor function: cannot invert without access to SK

Protocols with public key encryption

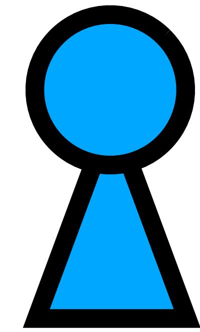
Goal: deliver a confidential message

Symmetric key

Email / chat

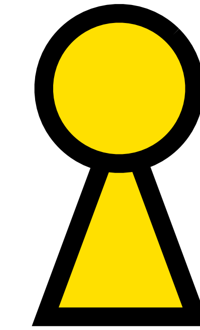


All-to-all:
$O(N^2)$ key
exchanges



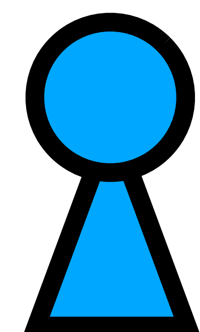
Generate public/private
key pair (PK,SK)

Annouce PK publicly
(on website, in newspaper, …)



Obtain PK

Send c = E(PK, msg)



Decrypt D(SK, c) = msg

O(N) keys in total

Overcoming fixed message sizes

Encryption E(PK, msg)

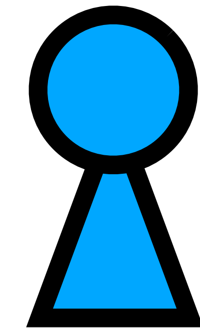
- Inputs
  - **Public** key PK
  - Message msg of *fixed size*
- Outputs: a cipher text *c* *same size as msg*

Like block ciphers, but there are not "modes" of public key encryption

Public key operations are *slooooow!*

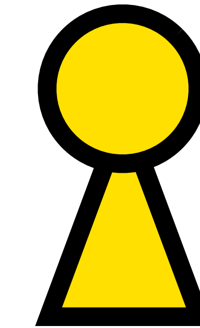
Symmetric key operations are fast

Hybrid encryption



Generate public/private key pair (PK,SK); publicize PK

Obtain PK

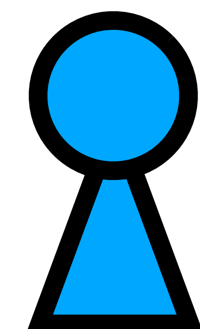


Generate *symmetric* key K

Symm key Compute $c_{msg} = e(K, msg)$

Public key Compute $c_K = E(PK, K)$ ***Now throw away K***

Send $c_K \| c_{msg}$



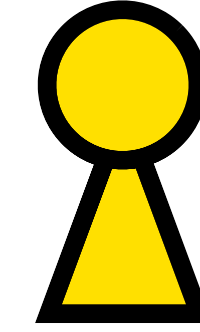
Decrypt $D(SK, c_K) = K$ *Public key*

Decrypt $d(K, c_{msg}) = msg$ *Symm key*

Hybrid encryption

Obtain PK

Generate *symmetric* key K



Compute c_{msg} = e(K, msg)

Compute c_K = E(PK, K)

Send $c_K \,||\, c_{msg}$

The easy key distribution of public key

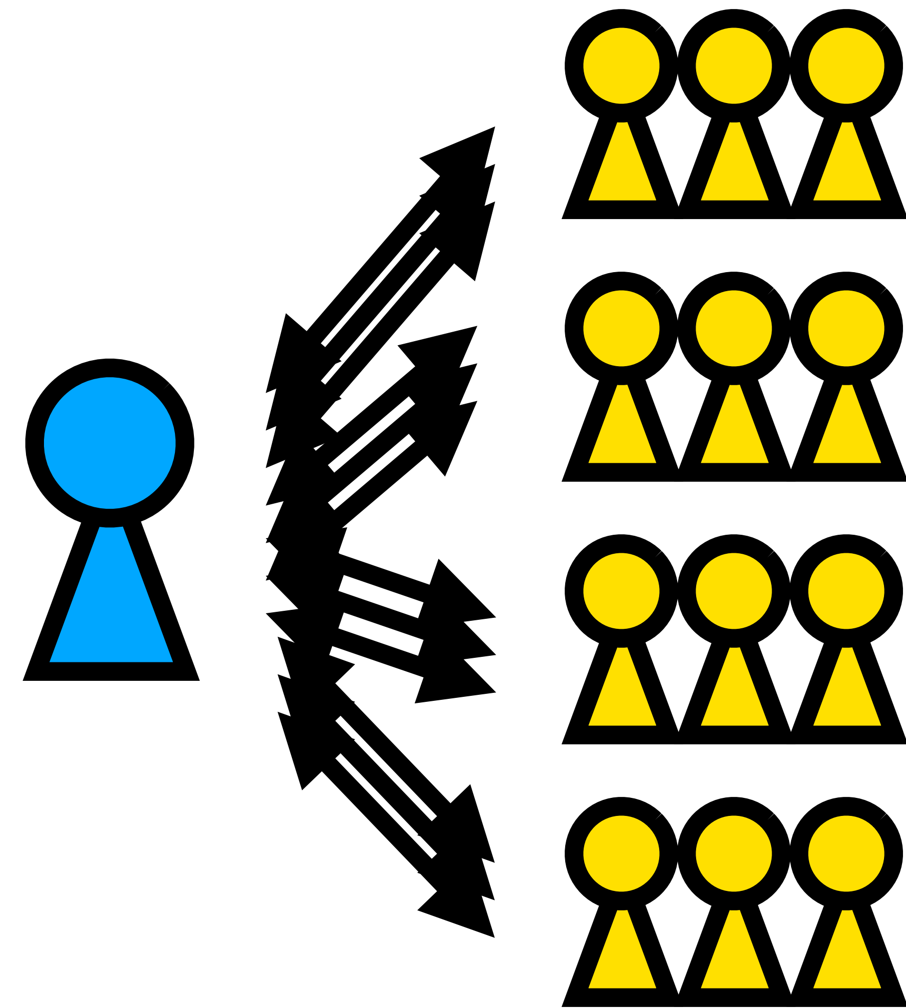
The speed and arbitrary message length of symmetric key

Protocols with public key cryptography

Goal: determine from whom a message came

Symmetric key

File downloads



One-to-many:
O(N) key
exchanges

Ideally, a user (blue) could post a message (e.g., sensitive documents or a kernel update), and then go offline

And downloaders (yellow) could subsequently infer the message's authenticity without having to have already established a pairwise key with the publisher

Digital signatures

A digital signature scheme comprises two algorithms

Signing function **Sgn(SK, m)**
- Inputs
  - **Secret** key SK
  - Fixed-length message
- Outputs: a *signature s*

This is a *randomized* algorithm
(nondeterministic output)

SK a.k.a. "Signing key"

Only one person can sign with a given (PK,SK) pair

Verification function **Vfy(PK, m, s)**
- Inputs
  - **Public** key PK
  - Message and signature
- Outputs: Yes/No if valid (m,s)

Deterministic algorithm

Anyone with the PK can verify

Digital signatures

A digital signature scheme comprises two algorithms

Signing Sgn(SK, m)
→ a *signature s*

Verification Vfy(PK, m, s)
→ Yes/No if valid (m,s)

Correctness

Vfy(PK, m, Sgn(SK, m)) = Yes

Security

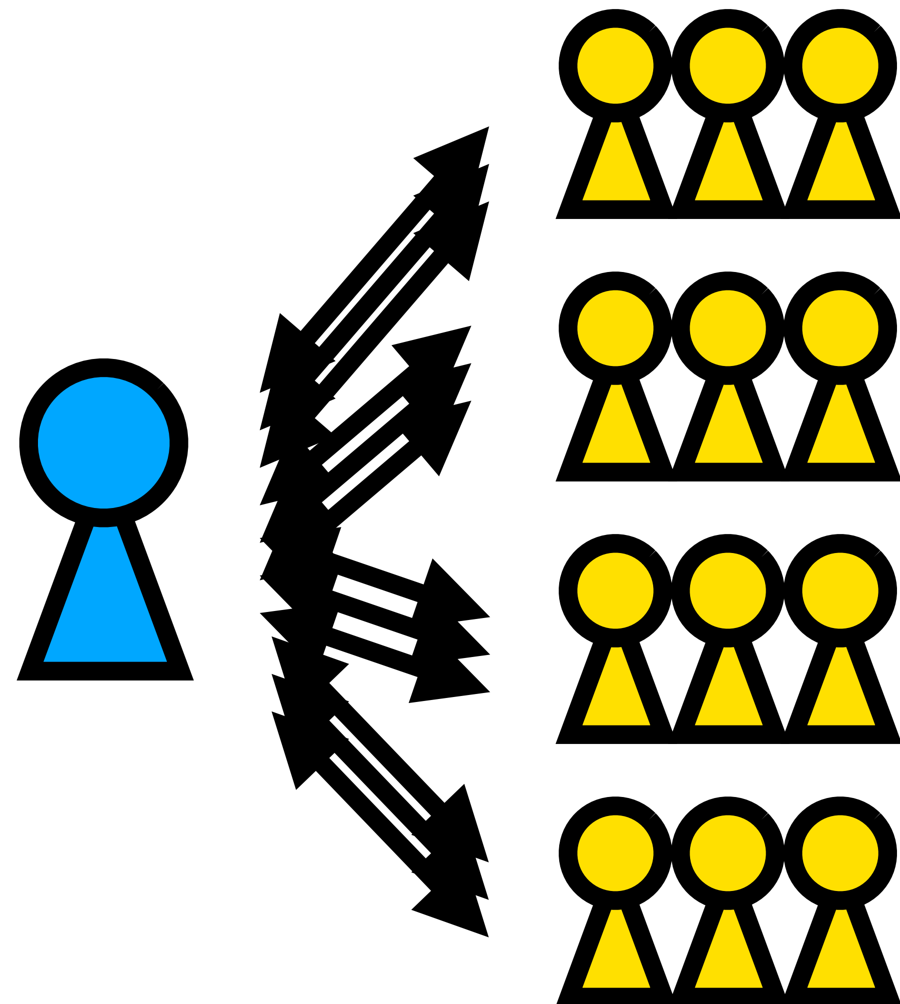
Same as with MACs: even after a chosen plaintext attack, the attacker cannot demonstrate an existential forgery

Protocols with digital signatures

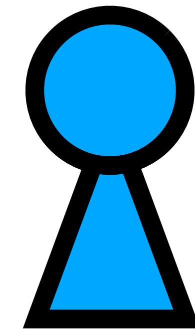
Goal: determine from whom a message came

Symmetric key

File downloads



One-to-many:
O(N) key
exchanges



Generate public/private
key pair (PK,SK)

Annouce PK publicly
(on website, in newspaper, …)

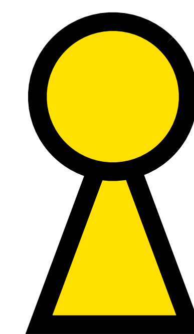
Compute sig = Sgn(SK, msg)

Publish msg || sig

can now go offline!

Obtain PK, msg || sig

Vfy(PK, msg, sig)



Digital signature properties

Authenticity

Bob can prove that a message signed by Alice is truly from Alice (even without a *pairwise* key)

Integrity

Bob can prove that no one has tampered with a signed message

Non-repudiation

Once Alice signs a message, she cannot subsequently claim she did *not* sign that message

Do *handwritten* signatures at the end of a letter have these properties?

Authenticity

Bob can prove that a message signed by Alice is truly from Alice (even without a *pairwise* key)

Would require unforgeable handwritten signatures. This is the one property they sort of get

Integrity

Bob can prove that no one has tampered with a signed message

Would require having a signature that depended on each part in the body of the letter

Non-repudiation

Once Alice signs a message, she cannot subsequently claim she did not sign that message

Would require both of the above (unforgeable signature that depends on each part of letter)

Agenda

- Diffie Hellman Key Exchange
- Public Key Cryptography
- Certificates
  - How do we distribute public keys securely?
- Passwords

Review: Public-Key Cryptography

- Public-key cryptography is great! We can communicate securely without a shared secret
  - Public-key encryption: Everybody encrypts with the public key, but only the owner of the private key can decrypt
  - Digital signatures: Only the owner of the private key can sign, but everybody can verify with the public key
- What's the catch?

Problem: Distributing Public Keys

- Public-key cryptography alone is not secure against man-in-the-middle attacks
- Scenario
  - Alice wants to send a message to Bob
  - Alice asks Bob for his public key
  - Bob sends his public key to Alice
  - Alice encrypts her message with Bob’s public key and sends it to Bob
- What can Mallory do?
  - Replace Bob's public key with Mallory’s public key
  - Now Alice has encrypted the message with Mallory’s public key, and Mallory can read it!

Problem: Distributing Public Keys

Alice

Mallory

Bob

Generate PK_B, SK_B

Send PK_B

Send PK_M

Receive PK_M

Send $\{M\}_{PK_M}$

Decrypt $\{M\}_{PK_M}$

Send $\{M\}_{PK_B}$

Decrypt $\{M\}_{PK_B}$

Problem: Distributing Public Keys

- Idea: Sign Bob's public key to prevent tampering
- Problem
  - If Bob signs his public key, we need his public key to verify the signature
  - But Bob's public key is what we were trying to verify in the first place!
  - Circular problem: Alice can never trust any public key she receives
- You cannot gain trust if you trust nothing. You need a root of trust!
  - **Trust anchor**: Someone that we implicitly trust
  - From our trust anchor, we can begin to trust others

Trust-on-First-Use

- **Trust-on-first-use**: The first time you communicate, trust the public key that is used and warn the user if it changes in the future
    - Used in SSH, Whatsapp and a couple other protocols
    - Idea: Attacks aren't frequent, so assume that you aren't being attacked the first time you communicate
    - Also known as "**Leap of Faith**"

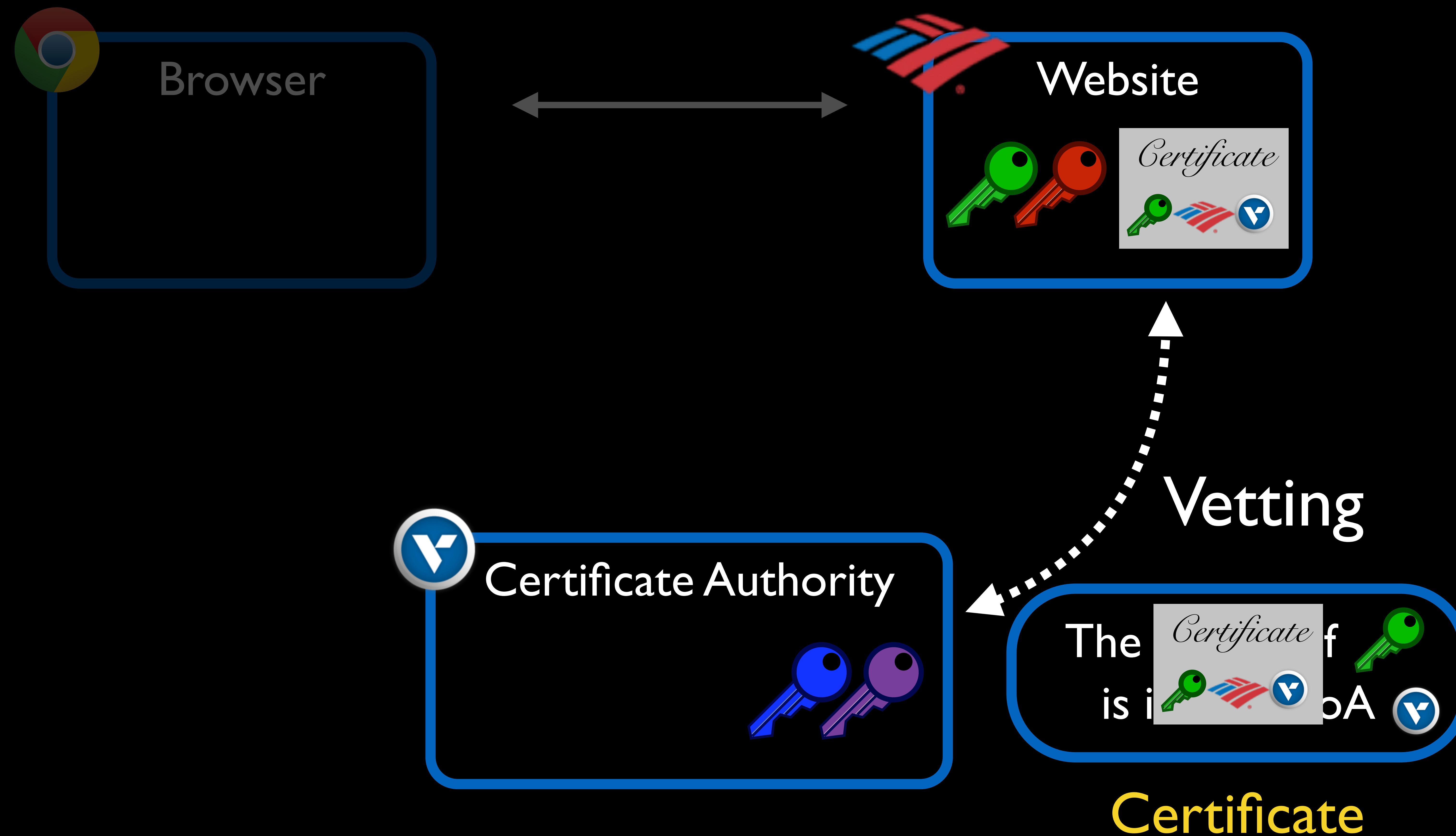
Certificates

- **Certificate**: A signed endorsement of someone's public key
  - A certificate contains at least two things: The **identity** of the person, and the **key**
- Recall: A signed message must contain the message along with the signature; you can't check the signature by itself!
- Scenario: Alice wants Bob's public key. Alice trusts EvanBot (PK_E, SK_E)
  - EvanBot is our trust anchor
  - If we trust PK_E, a certificate we would trust is
  - {"Bob's public key is PK_B"} message signature signed by SK_E

PUTTING IT ALL TOGETHER:
PUBLIC KEY INFRASTRUCTURE

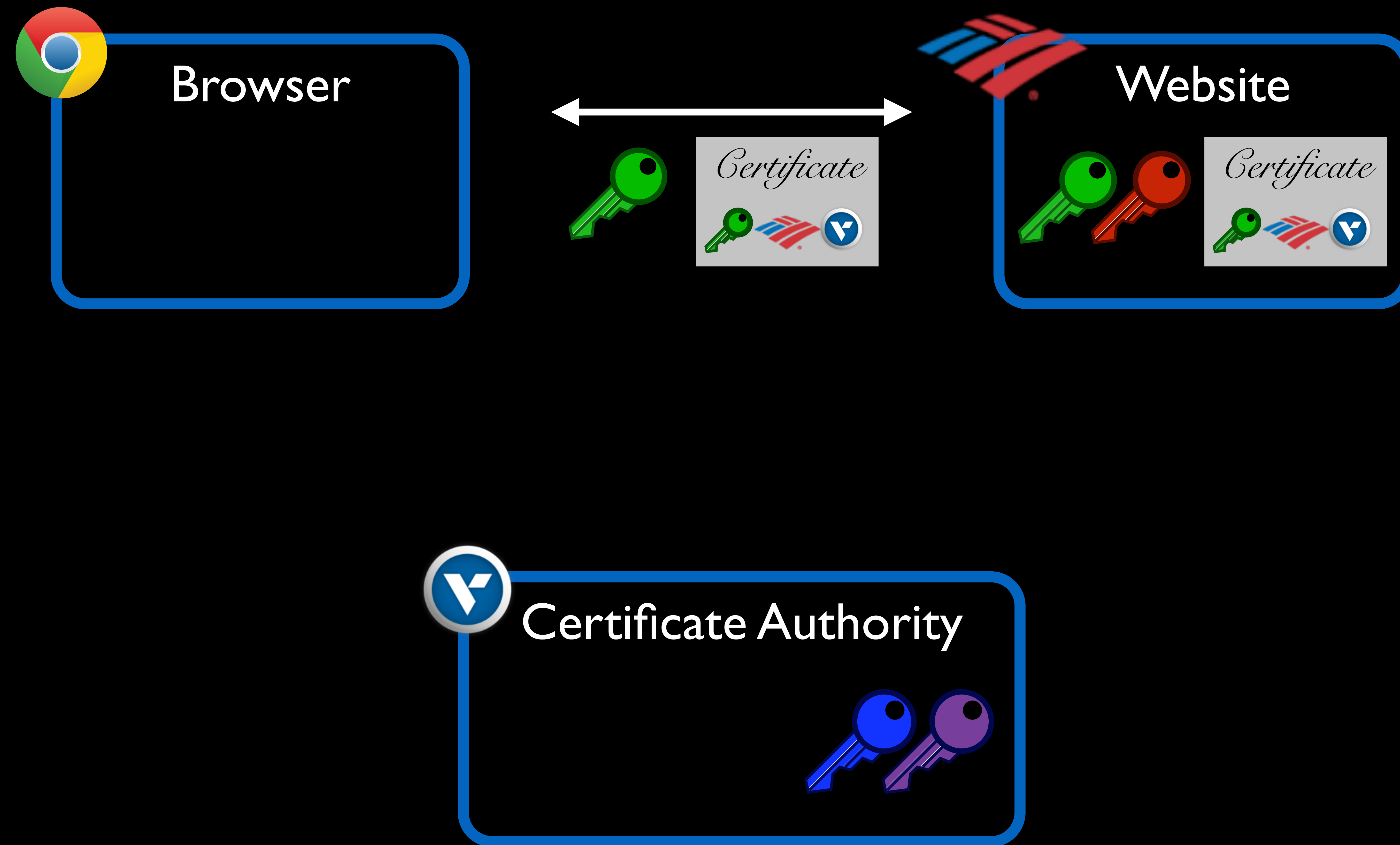
Public Key Infrastructures (PKIs)

How can users truly know with whom they are communicating?



Public Key Infrastructures (PKIs)

How can users truly know with whom they are communicating?



Verifying certificates

Root key store

Every device has one

Must not contain malicious certificates

Keychain Access

Click to unlock the System Roots keychain.

Search

Keychains
- login
- iCloud
- System
- System Roots

Category
- All Items
- Passwords
- Secure Notes
- My Certificates
- Keys
- Certificates

Symantec Class 1 Public Primary Certification Authority - G4
Root certificate authority
Expires: Monday, January 18, 2038 at 6:59:59 PM Eastern Standard Time
This certificate is valid

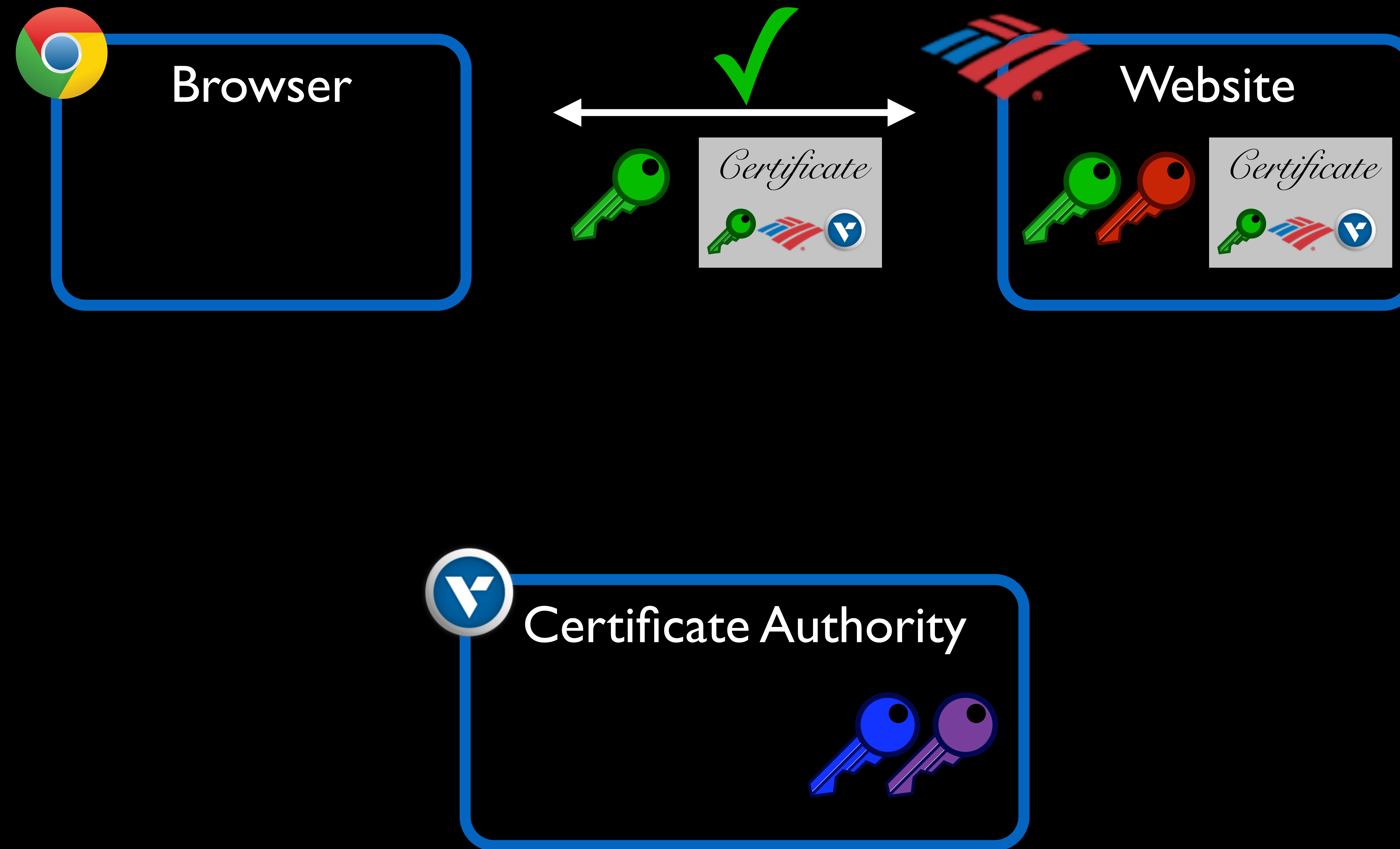
Name	Kind	Expires	Keychain
Starfield Class 2 Certification Authority	certificate	Jun 29, 2034, 1:39:16 PM	System Roots
Starfield Root Certificate Authority - G2	certificate	Dec 31, 2037, 6:59:59 PM	System Roots
Starfield Services Root Certificate Authority - G2	certificate	Dec 31, 2037, 6:59:59 PM	System Roots
StartCom Certification Authority	certificate	Sep 17, 2036, 3:46:36 PM	System Roots
StartCom Certification Authority	certificate	Sep 17, 2036, 3:46:36 PM	System Roots
StartCom Certification Authority G2	certificate	Dec 31, 2039, 6:59:01 PM	System Roots
Swisscom Root CA 1	certificate	Aug 18, 2025, 6:06:20 PM	System Roots
Swisscom Root CA 2	certificate	Jun 25, 2031, 3:38:14 AM	System Roots
Swisscom Root EV CA 2	certificate	Jun 25, 2031, 4:45:08 AM	System Roots
SwissSign CA (RSA IK May 6 1999 1		26, 2031, 6:27:41 PM	System Roots
SwissSign Gold CA - G2	certificate	Oct 25, 2036, 4:30:35 AM	System Roots
SwissSign Platinum CA - G2	certificate	Oct 25, 2036, 4:36:00 AM	System Roots
SwissSign Silver CA - G2	certificate	Oct 25, 2036, 4:32:46 AM	System Roots
Symantec Class 1 Public Primary Certification Authority - G4	certificate	Jan 18, 2038, 6:59:59 PM	System Roots
Symantec Class 1 Public Primary Certification Authority - G6	certificate	Dec 1, 2037, 6:59:59 PM	System Roots
Symantec Class 2 Public Primary Certification Authority - G4	certificate	Jan 18, 2038, 6:59:59 PM	System Roots
Symantec Class 2 Public Primary Certification Authority - G6	certificate	Dec 1, 2037, 6:59:59 PM	System Roots
Symantec Class 3 Public Primary Certification Authority - G4	certificate	Dec 1, 2037, 6:59:59 PM	System Roots
Symantec Class 3 Public Primary Certification Authority - G6	certificate	Dec 1, 2037, 6:59:59 PM	System Roots
SZAFIR ROOT CA	certificate	Dec 6, 2031, 6:10:57 AM	System Roots
T-TeleSec GlobalRoot Class 2	certificate	Oct 1, 2033, 7:59:59 PM	System Roots
T-TeleSec GlobalRoot Class 3	certificate	Oct 1, 2033, 7:59:59 PM	System Roots
TC TrustCenter Class 2 CA II	certificate	Dec 31, 2025, 5:59:59 PM	System Roots
TC TrustCenter Class 3 CA II	certificate	Dec 31, 2025, 5:59:59 PM	System Roots
TC TrustCenter Class 4 CA II	certificate	Dec 31, 2025, 5:59:59 PM	System Roots
TC TrustCenter Universal CA I	certificate	Dec 31, 2025, 5:59:59 PM	System Roots
TC TrustCenter Universal CA II	certificate	Dec 31, 2030, 5:59:59 PM	System Roots
TC TrustCenter Universal CA III	certificate	Dec 31, 2029, 6:59:59 PM	System Roots

Copy

210 items

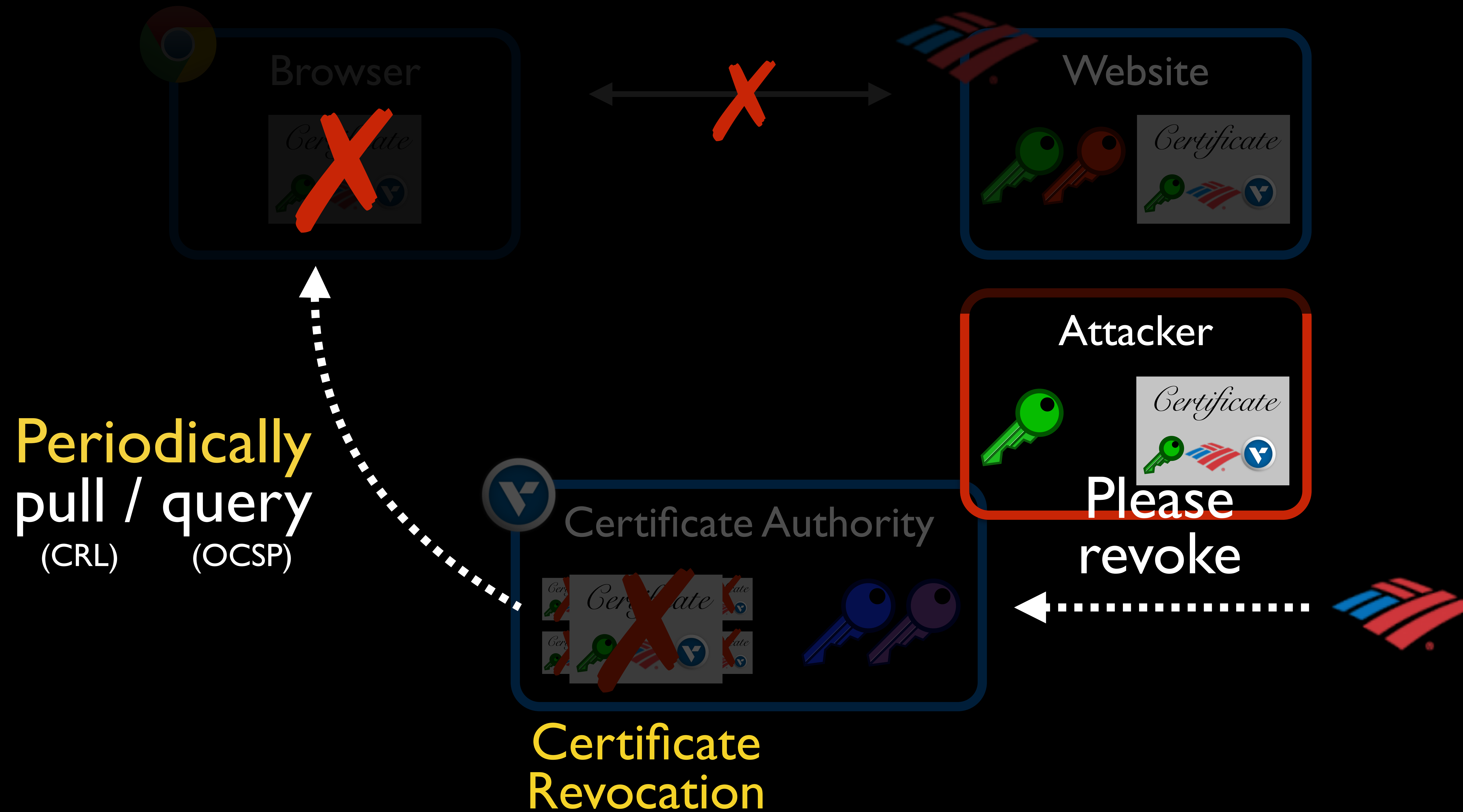
Public Key Infrastructures (PKIs)

How can users truly know with whom they are communicating?



Certificate revocation

What happens when a certificate is no longer valid?



Certificate revocation

is a critical part of any PKI

Administrators must revoke and reissue as quickly as possible

Browsers/OSes should obtain revocations as quickly as possible

HASH FUNCTION APPLICATION
STORING PASSWORDS

THREAT MODEL

- Attacker can eventually gain access to the *hard drive* where (some version of) the passwords are stored long-term
- But attacker does *not* gain access to memory (where raw passwords might be stored while processing)
- Attacker gets as much prep time as they want, but not unlimited amounts of storage
- Goal of the attacker: recover passwords within some window of time

FAILED IDEA #1: STORE THE PASSWORDS

username : password

- Attacker can eventually gain access to the *hard drive* where (some version of) the passwords are stored long-term

The attacker trivially gains access to the passwords

FAILED IDEA #2: STORE ENCRYPTED PASSWORDS

$$username : E(K, IV, password), IV$$

- Attacker can eventually gain access to the *hard drive* where (some version of) the passwords are stored long-term

This can work if the key is not stored on the hard drive

But if the key is stored on the hard drive, then it is trivial for the attacker to recover

FAILED IDEA #3: STORE HASHED PASSWORDS

username : H(password)

- Attacker can eventually gain access to the *hard drive* where (some version of) the passwords are stored long-term

Problem 1: many users use the same password

Most common *H(password)* = most common password

1. 123456
2. password
3. 12345678
4. qwerty
5. 123456789
6. 12345
7. 1234
8. 111111
9. 1234567
10. dragon

Problem 2: attacker gets prep time

They can precompute hashes

(H(123456), H(password), ...)

More compact representation of this is a rainbow table

RAINBOW TABLES

$$username : H(password)$$

- Goal: compact storage of hashes of many passwords

$$\texttt{aaaaaa} \xrightarrow[H]{} \texttt{281DAF40} \xrightarrow[R]{} \texttt{sgfnyd} \xrightarrow[H]{} \texttt{920ECF10} \xrightarrow[R]{} \texttt{kiebgt}$$

Hash *"Reduction"* $kiegbt = R(H(R(H(aaaaaa))))$

A "reduction" function is simply a function that takes a hash's output as its input and outputs a potential input (in this case, a 6-letter password)

Only store the beginning of this chain (aaaaaa) and the end (kiebgt)

RAINBOW TABLES

username : H(password)

$$\texttt{aaaaaa} \xrightarrow[H]{} \texttt{281DAF40} \xrightarrow[R]{} \texttt{sgfnyd} \xrightarrow[H]{} \texttt{920ECF10} \xrightarrow[R]{} \texttt{kiebgt}$$

Only store the beginning of this chain (aaaaaa) and the end (kiebgt)

Do this for many initial seed inputs (bbbbbb, password, 123456, etc.)

Given H(password)

$x = R(H(password))$

Is x one of the end values (e.g., kiebgt)?

If so, then the password must have been one of the passwords in the chain

If not, then $y = H(x); x = R(y)$ *and try again*

Give up after some maximum number of tries

FAILED IDEA #4: STORE SALTED HASHED PASSWORDS

$$username : H(salt \mid password), salt$$

- Remember: *small changes to the input leads to large, unpredictable changes in the output*

Good news: Rainbow tables don't work anymore

Bad news: Can still try a dictionary attack against a given user because **hash functions are *very efficient to compute***

*Ideally we would have a very **slow** hash function*

How can we create a slow hash function out of a fast hash function?

HOW PASSWORDS ARE STORED

username : H^k(salt | password), salt

- $H^k = H(H(H(\ldots H(x)\ldots)))$
- Compute the hash of the hash of the hash of the…

H is a fast hash function; H^k is a slow one!

This is how passwords are stored in Linux today

Recall: Given H(password), it is infeasible to recover password,

So what does it mean if a website can email you your password?